XV Anidis Conference – Padova 2013

**Dipl.-Ing. Karl Burgmann – Prof. Dr.-Ing. Casimir Katz, SOFiSTiK AG**

# On the correct treatment of the directional superposition of response spectra

**Summary**
General analysis of structures for earthquake loading is governed by two methods. Besides the most general direct integration of a real or artificial acceleration history including all possibilities of non linear effects the usage of response spectra has become the most popular method. However there are two points which are not dealt properly in many cases. They are the superposition of the results from several Eigenforms and the superposition of the possible directions.

## 1. Introduction

The method of response spectra is described in detail in most textbooks [3,4,5]. Summarized in a few words a single mass oscillator is analysed by a predetermined acceleration curve and the maximum response is evaluated. Plotting this response over the natural oscillation period in a diagram, one obtains a curve that has been defined in the standards as an envelope as the corresponding response spectra function.

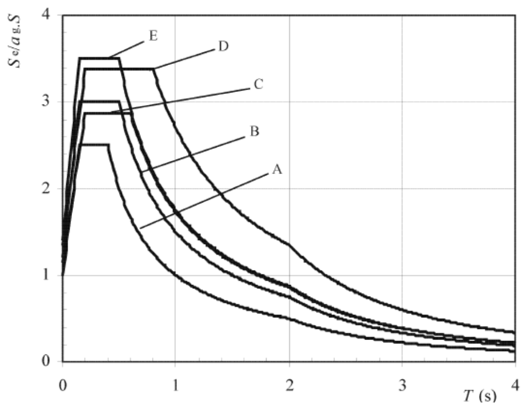


**Fig. 1:** Response spectra from the Eurocode

The analysis process is now to pick for each natural frequency of the structural system a modal response of the spectrum. The modal response is not only valid for the acceleration; the same factor applies for the displacements and the resulting forces. Unfortunately in recent times the method of equivalent forces has become very popular, which obscures the essential background of the process. Here, only the so-called modal forces are considered, such as shown by Meskouris et. al [3]. These are forces directly related to the deformation of the mode.

The apparent disadvantage of the response spectrum method is that the information about the time of the maximum is lost. One has to use statistical methods to combine the results of different response values, but the excitation is not deterministic at all, so this is natural and not a drawback.

## 2. Superposition of the Eigen forms

Many design codes still use the method of the square root of the sum of the squares:

$$q_j = \sqrt{\sum_i q_{ij}^2}$$

But it is common knowledge, that the complete quadratic combination (CQC) gives much better results.

$$q = \sqrt{\sum \sum q_i \cdot \rho_{ij} \cdot q_j}$$

$$\rho_{ij} = \frac{8\sqrt{d_i d_j}\,(d_i + r \cdot d_j) \cdot r^{\frac{3}{2}}}{(1 - r^2)^2 + 4 d_i d_j\, r \cdot (1 + r^{2)} + 4(d_i^2\, d_j^2) r^2}$$

The first method SRSS is a special case of the second. Significant differences arise when the frequencies of two mode shapes are very close or even identical. It must be pointed out that the knowledge of the damping for the calculation of the coupling coefficients is essential. Without any damping, the CQC method equals the SRSS method with one exception: the same natural frequencies have off-diagonal elements, which are formally undefined, but obtained by a limiting process with the value of 1.0.

Looking closer to these formulas all results are positive. This is no surprise, because the direction of excitation was indeed lost in the response spectrum, i.e. the results must be introduced with changing sign to other superposition or design. The earthquake may come from any direction. But the stress in a structure is given not only by one cross-sectional force. There are normal forces and moments and looking at a very simple framework, one has to recognize that these are not identical in their direction of action:

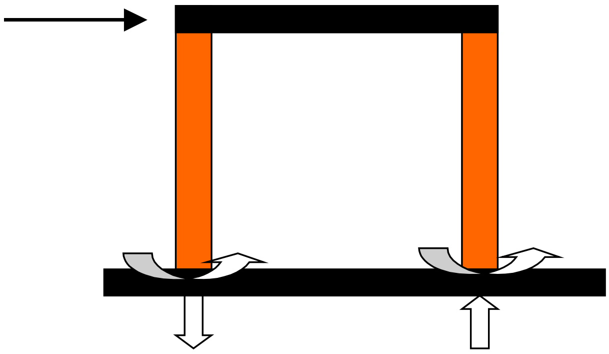

**Fig. 2:** Forces and Moments for a frame with horizontal loading.

Although the loading produces the same moments, the normal forces have opposite directions. This may be at first glance important only for building materials with different strengths in the tension and compression area. But the effect is important for standard cases as well, which will be shown in the following example with double eigenvalues. What we need and get form the superposition is a maximum (positive) moment. But the corresponding normal force with the correct sign and value is not obtained. It is neither economic nor on the save side to use the absolute maximum values of all forces and to combine them with alternating signs.

This dilemma is solved by a very simple trick [6]. Writing for example the SRSS formula not as root but as a linear combination of modal vectors involved, which represent the base of a vector space:

$$SUM = \sum_i f_i \cdot q_i \ ; \ f_i = \frac{q_i}{\sqrt{\sum_j q_j^2}}$$

As one can easily see, this formula yields the same maximum value of the target, but now all signs of the corresponding internal forces are maintained and as the modal forces to a mode shape have a distinct and unique sign, you get a combination of forces and moments that is consistent.

The procedure for the CQC-Method is only slightly more enhanced:

$$SUM = \sum_i f_i \cdot q_i \ ; \ f_i = \frac{\sum_j \rho_{ij} q_j}{\sqrt{\sum_i \sum_j q_i \rho_{ij} q_j}}$$

# 3. Superposition of directions

Similarly, one can proceed with the directions. Since all phase information is lost, there is the actual task of finding the least favourable combination of directions of the earthquake. In the Eurocode you will find the formulas (4.18) and (4.19):

a) $E_{\mathrm{Ed}x}$ "+" $0{,}30 E_{\mathrm{Ed}y}$

b) $0{,}30 E_{\mathrm{Ed}x}$ "+" $E_{\mathrm{Ed}y}$

Other references indicate that one should always use 30% or even 40 % across the main direction of the earthquake. Although it is clear that an earthquake does not have a distinct direction the specified base acceleration is a maximum value in any direction, if one misuses the above formula to add just a second transverse acceleration one enters a never ending recursion always adding a transverse component. A correct approach would be to apply the acceleration in any direction with the components sinΘ and cosΘ. If we consider the response in two orthogonal directions $S_x$ and $S_y$ the total response is given by:

$$S = S_x \cdot \sin\Theta + S_y \cdot \cos\Theta \;\; ; \;\; \max S \Rightarrow \tan\Theta = \frac{S_x}{S_y}$$

$$\max S = S_x \cdot \frac{S_x}{\sqrt{S_x^2 + S_y^2}} + S_y \cdot \frac{S_y}{\sqrt{S_x^2 + S_y^2}} = \sqrt{S_x^2 + S_y^2}$$

If the correct solution is so easy, why is it not used in general? The old German design code DIN 4149 specified in chapter 6.2.4 „Combination of Response due to the components of the earthquake action“:

> (2) The combination of the horizontal components of the seismic action may be considered in the following manner: The internal forces and displacements of the structure must be determined separately for each horizontal component, the maximum value of each force has to be calculated in this case as the square root of the sum of the two horizontal components.
>
> (3) As an alternative to paragraph (2) a combination of the horizontal component of earthquake excitation internal forces are calculated using the following two combinations: Ex ⊕ 0.30 Ey ; 0.30 Ex ⊕ Ey.

The second rule would indeed introduce a sign, but the formulas described in the previous paragraph make it possible to combine the sign mathematically correct.

# 4. Example

To demonstrate the method a cantilever column is examined. All bending eigenvalues occur twice, the corresponding Eigen forms are orthogonal, but an arbitrary direction. Depending on the strategy of the eigenvalue solver you will therefore get different directions. The inverse iteration used here starts with a random vector. Thus the mode shapes have been obtained approximately but not exactly in the diagonal directions:

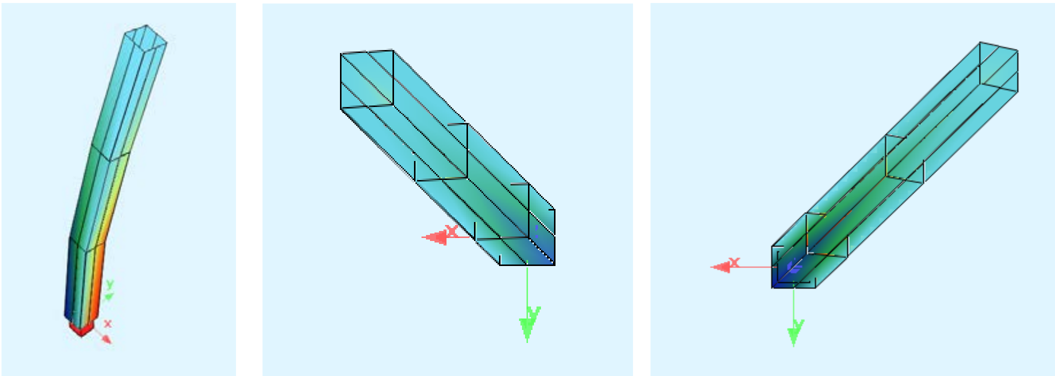


**Fig. 3:** Eigenforms for a double Eigenvalue of a cantilever column

The exact dimensions and loads are not essential and therefore not further specified. Even the damping may be neglected in this case. With the values chosen, the two Eigen modes yielded orthogonal, but random modal moments My and Mz:

Eigenform 1   My = -7136; Mz = +2707
Eigenform 2   My = -2707; Mz = +7136

Now the two orthogonal accelerations are applied in two exactly diagonal directions (+45 degrees and -45 degrees). The results for the maximum moment My are obtained with an associated signed transverse bending:

| | | | | |
|---|---|---|---|---|
| Acceleration diagonal to the upper left: | My | 13.15 | Mz | -13.15 |
| Acceleration diagonal to the lower left: | My | 13.15 | Mz | 13.15 |

It has to be noted that this symmetric result is obtained only with the CQC method. When SRSS method is used to combine the eigenvalues, significant deviations remain from the non-symmetric Eigen forms.

Here the task is not to find the maximum bending moment My. This is obtained if the acceleration is exactly in the direction of the local z-axis. But then the transverse acceleration and hence the stress are zero! So if the two directions are added correctly, the components of the bending moment Mz must cancel, which is possible only if the sign is considered as described in the previous paragraph:

| | | | | |
|---|---|---|---|---|
| Max My from all directions: | My | 18.60 | Mz | 0.00 |
| Max Mz from all directions: | My | 0.00 | Mz | 18.60 |

The same result is obtained for any orthogonal directions and even for using the SRSS method for the modal superposition. Using the simplified method one would obtain 13.15*1.3 and 13.15*0.7. Using the SRSS for the modal superposition generates pure random noise, some values even larger than the correct ones. All these values are not really acceptable.

The remaining question about that result is that we do not know if the maximum moment is the critical effort to be designed for. What happens for the diagonal directions? Well this is not a question of the procedure, but of the result requested. If we are interested in the maximum stress at the corner we have to build a new target as a combination of the two moments, or more general as a linear combination of any forces:

$$S = \sum a_i \cdot S_i = \frac{1}{A} \cdot N + \frac{1}{W_y} \cdot M_y + \frac{1}{W_z} \cdot M_z$$

For any linear superposition this extension is straightforward extendable to the procedure given above, we just obtain different factors. For this example we obtain the two moments with the values as stated above. So we may have four excluding sets of consistent forces to be introduced with a factor of -1 or +1 in the superposition.

This extended superposition scheme has been called a polytope superposition [8] and is usable for all difficult situations where the efforts are not independent. Examples are normal force and moment in a concrete column; the maximum effort has to be searched in the direction orthogonal to the lines of constant reinforcements:

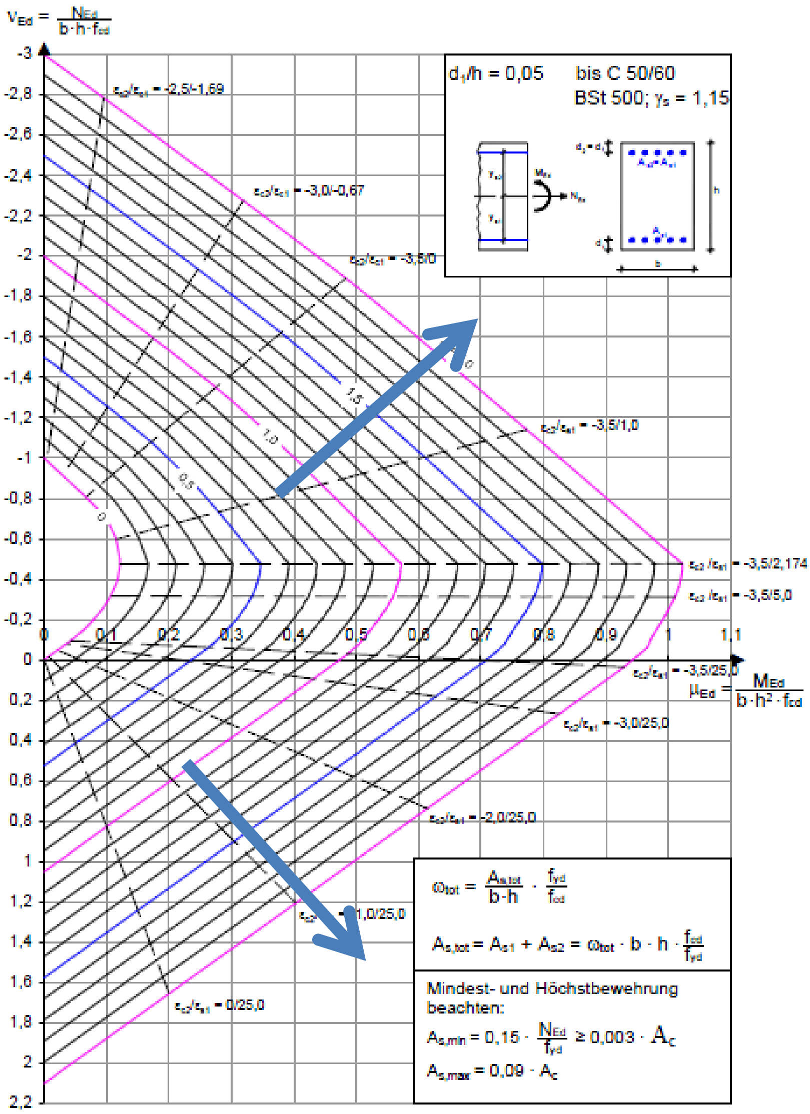


**Fig. 4:** Interaction diagram for a concrete column [9]

# 5. The case of transient analysis

If we switch to a transient analysis we have to use a distinct acceleration taken from measurements or created by a random process like SIMQKE [10]. But in this case we do not need only to generate a sufficient set of samples (clause (4a) of 3.2.3.1.2 requires only a minimum of 3 samples), but we have also to account for the directional diversity. It is a really bad idea to apply the same acceleration history in two coordinate directions; this is explicitly forbidden in clause (2) of 3.2.3.1.1 in the EN 1998-1. Thus there are several possibilities to account for the general requirements of matching the $a_g$-criteria:

- Following the outlined maximisation of the previous paragraph one possible solution would be to apply the selected sample in various directions. We then have to combine the results in an exclusive way.
- We could apply the samples in two orthogonal directions and apply the SRSS superposition for the final result.
- We can apply just two independent samples in two orthogonal directions. To cover the one directional example, both samples have to be based on the full value of the base acceleration $a_g$. The unfavourable superposition of two peaks at the same time is not as important as it will be for a very short time only.

.As the design code uses transient analysis only in the term of a nonlinear transient analysis it is only the third possibility which could be applied. A directional superposition is not requires in that case.

# 6. The case of push-over analysis

The push-over analysis has two important draw backs here. Although there are some extensions for multimodal solutions it is intended only for a single Eigen form or a single linear combination of Eigenforms. Second the static push-over curve has to be selected in the same direction. The method is restricted to systems where clear directions of deformation are present.

# 7. Summary

The usage of the improper superposition rule used in nearly all design codes, combined with the unsatisfactory SRSS method for modal superposition is not the state of the art. The demonstrated modification in the superposition makes it possible to obtain consistent results not only for the modal superposition, but also for the directional superposition. Thus it should be avoided to take the rule of the 30% loading in transverse direction as the proper or even unique solution of the problem.

Further it is a question if the treatment of the problem with equivalent loadings is helpful to understand the mathematical true nature of the problem.

# 8. Literature


[1] Eurocode 8, Part 1-6, European Standard 2010

[2] DIN 4149, Bauen in deutschen Erdbebengebieten, 2004

[3] K.Meskouris, K-G. Hinzen, C. Butenweg, M. Mistler,
Bauwerke und Erdbeben, 2.Auflage Vieweg, 2007

[4] R. Flesch, Baudynamik praxisgerecht, Bauverlag Wiesbaden, 1993

[5] R.W. Clough, J. Penzien Dynamics of Structure, 3rd Edition,
Computers & Structures, Inc., Berkeley, 2003

[6] C. Katz, Handbuch DYNA, SOFiSTiK Oberschleißheim, 2003

[7] C. Katz, Anmerkung zur Überlagerung von Antwortspektren
D-A-CH-Mitteilungsblatt der DGEB im Bauingenieur March 2009

[8] R. Tautz, S. Maly, C. Katz, Combination of Actions for all Established European Codes in Structural Engineering with Polytope Superposition
IABSE-fib Conference Dubrovnik 2010: Codes in Structural Engineering - Developments and Needs for International Practice

[9] M.Rößler, public.beuth-hochschule.de/~roeslerm/Hilfsmittel.pdf

[10] http://nisee.berkeley.edu/documents/SWSC/SIMQKE1.zip